

Section P.1 Graphs and Models

Ex.1 Find any intercepts:  $y = \frac{2 - \sqrt{x}}{5x + 1}$

Find x-intercept: let  $y = 0$

$$0 = \frac{2 - \sqrt{x}}{5x + 1} \quad 5x + 1 \neq 0$$

$$(5x + 1) \cdot 0 = (5x + 1) \left( \frac{2 - \sqrt{x}}{5x + 1} \right)$$

$$\sqrt{x} + 0 = 2 - \sqrt{x} + \sqrt{x}$$

$$(\sqrt{x})^2 = 2^2$$

$$x = 4$$

$$\underline{\underline{(4, 0)}}$$

Find y-intercept:  $x = 0$

$$y = \frac{2 - \sqrt{0}}{5 \cdot (0) + 1}$$

$$y = \frac{2}{1}$$

$$y = 2$$

$$(0, 2)$$

Ex.2 Test for symmetry with respect to each axis and the origin:  $y = \frac{x^2}{x^2 + 1}$

let  $f(x) = y$

Consider

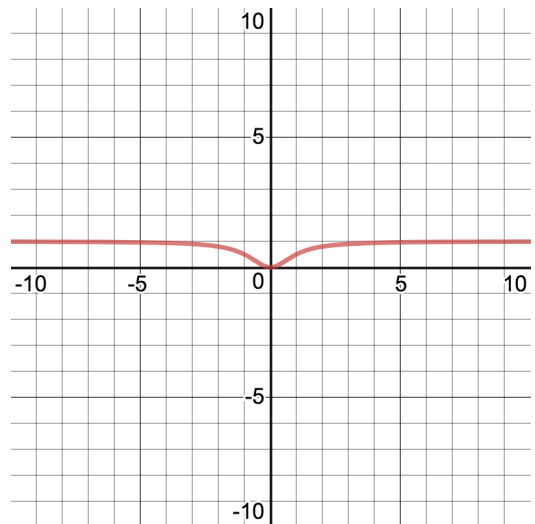
$$f(-x) = \frac{(-x)^2}{(-x)^2 + 1}$$

$$f(x) = \frac{x^2}{x^2 + 1}$$

$$f(-x) = f(x)$$

Even function

Symmetric about the y-axis



Elimination  
"Get rid of y's"

Ex.3 Find the points of intersection:

$$\begin{cases} 3x - 2y = -4 \\ 4x + 2y = -10 \end{cases}$$

$$\begin{array}{r} 3x - 2y = -4 \\ + 4x + 2y = -10 \\ \hline 7x = -14 \\ \frac{7x}{7} = \frac{-14}{7} \\ x = -2 \end{array}$$

Point is  $(-2, -1)$ :

$$\begin{array}{r} 3x - 2y = -4 \\ 3x + 4 = 2y \\ \hline \frac{3x + 4}{2} = \frac{2y}{2} \end{array}$$

$$y = \frac{3}{2}x + 2$$

$y_1$

Find y: use substitution

$$\begin{array}{r} 4x + 2y = -10 \\ 4(-2) + 2y = -10 \\ -8 + 2y = -10 \\ -8 + 2y + 8 = -10 + 8 \\ \frac{2y}{2} = \frac{-2}{2} \\ y = -1 \end{array}$$

$$\begin{array}{r} 4x + 2y = -10 \\ \frac{2y}{2} = \frac{-4x - 10}{2} \end{array}$$

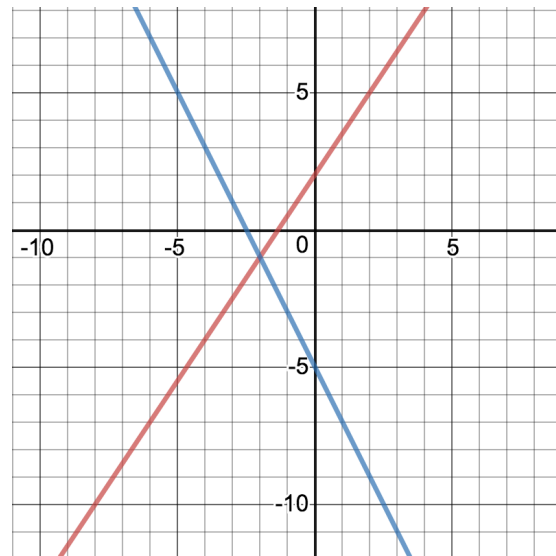
$$y = -2x - 5$$

$y_2$

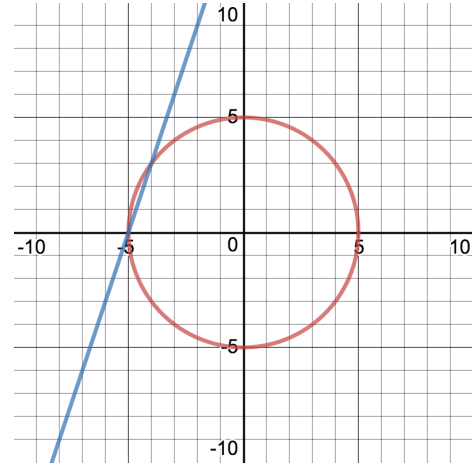
check:

$$\begin{array}{r} 3(-2) - 2(-1) = -4 \\ -6 + 2 = -4 \\ -4 = -4 \\ \text{TRUE!} \end{array}$$

$$\begin{array}{r} 4(-2) + 2(-1) = -10 \\ -8 - 2 = -10 \\ -10 = -10 \\ \text{TRUE!} \end{array}$$



Ex.4 Find the points of intersection:  $\begin{cases} x^2 + y^2 = 25 \\ -3x + y = 15 \end{cases}$



Solve by substitution.

Solve for  $y$ :  $-3x + y = 15$

$$-3x + y + 3x = 3x + 15$$

$$y = 3x + 15$$

SDWK

In  $x^2 + y^2 = 25$ , replace  $y$ .

$$x^2 + (3x + 15)^2 = 25$$

$$x^2 + 9x^2 + 90x + 225 = 25$$

$$10x^2 + 90x + 225 - 25 = 25 - 25$$

$$10x^2 + 90x + 200 = 0$$

$$\frac{1}{10}(10x^2 + 90x + 200) = \frac{1}{10} \cdot 0$$

$$x^2 + 9x + 20 = 0$$

$$(x + 4)(x + 5) = 0$$

Either

$$x + 4 = 0, \text{ or } x + 5 = 0$$

$$x = -4, \text{ or } x = -5$$

$$\begin{aligned} (3x + 15)^2 &= (3x + 15)(3x + 15) \\ &= 9x^2 + 45x + 45x + 225 \\ &= 9x^2 + 90x + 225 \end{aligned}$$

check:  $(-4, 3)$

$$(-4)^2 + (3)^2 = 25$$

$$16 + 9 = 25$$

$$25 = 25 \checkmark$$

$$\rightarrow (-4) + (3) = 15$$

$$-4 + 3 = 15$$

$$15 = 15 \checkmark$$

check:  $(-5, 0)$

$$(-5)^2 + (0)^2 = 25$$

$$25 = 25 \checkmark$$

$$-3(-5) + (0) = 15$$

$$15 = 15 \checkmark$$

Find  $y$ :

use  $y = 3x + 15$

If  $x = -4$

$$y = 3(-4) + 15$$

$$y = -12 + 15$$

$$y = 3$$

If  $x = -5$

$$y = 3(-5) + 15$$

$$y = -15 + 15$$

$$y = 0$$

The solution set is

$$\{(-4, 3), (-5, 0)\}$$